NWFP, PUBLIC SERVICE COMMISSION, PESHAWAR.

COMPETATIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICE, 2008.

APPLIED MATHEMATICS, PAPER-1

Time: 3 hours. **Max Marks:** 100

Note: Attempt only **FIVE** questions selecting at least **TWO** questions from each section. Each part carries **10** marks.

SECTION-A

Q.1. (a). Prove that vectors $|\underline{a}|\underline{b}+|\underline{b}|\underline{a}$ and $|\underline{a}|\underline{b}-|\underline{b}|\underline{a}$ are perpendicular to each other. Also prove that

$$|\underline{a}|^2+|\underline{b}|^2+|\underline{c}|^2+|\underline{a}+\underline{b}+\underline{c}|^2=|\underline{a}+\underline{b}|^2+|\underline{b}+\underline{c}|^2+|\underline{c}+\underline{a}|^2$$

(b). If the plane containing $\underline{a}$ and $\underline{b}$ is normal to the plane containing $\underline{c}$ and $\underline{d}$; then prove that

$$(\underline{a}\times\underline{b})\bullet(\underline{c}\times\underline{d})=0$$

Q.2. (a). Prove that the gradient of a scalar function Φ is the directional derivative of Φ perpendicular to the level Surface at p(x, y, z).

(b). If $\underline{r}=x\underline{i}+y\underline{j}+z\underline{k}$ and $\underline{a}$ is a constant vector, prove that $\underline{\nabla}\times(\underline{a}\times\underline{r})=2\underline{a}$

Q.3. (a). The upper end of a uniform ladder rests against a rough vertical wall and the lower end on a rough horizontal plane, the coefficient of friction in both the cases being 1/3. Prove that if the inclination of the ladder to the vertical wall is $\tan^{-1}(\frac{1}{2})$, a weight equal to that of the ladder can not be attached to it at a point more than 9/10 of the distance from the foot of it without destroying the equilibrium

(b). Find the center of mass of the surface generated by the revolution of the arc of the parabola lying between the vertex and the latus rectum, about the x-axis.

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SECTION-B

Q.4. (a). Coplanar forces (X_r, Y_r) act at point (x_r, y_r), where r = 1, 2,..., n. If each force is rotated about its point of application through an angle θ, show that the resultant passes through a fixed point for all values of θ.

(b). A triangular lamina ABC, right angled at A, rests with its plane vertical, and with the sides AB, AC supported by smooth pegs D, E in a horizontal line. Prove that the inclination θ of AC to the horizontal is given by

$$AC\cos\theta - AB\sin\theta = 3DE\cos 2\theta$$

Q.5. (a). Discuss the motion of a particle moving in a straight line, if it start from rest at a distance a from a point O and moves with an acceleration equal to μ-times its distance from O.

(b). A particle of mass m moves under the influence of the force $\underline{F} = a\left(\sin\omega t\underline{i} + \cos\omega t\underline{j}\right)$. If the particle is initially at rest at the origin, prove that the work done up to time t is given by $\frac{a^2}{m\omega^2}(1-\cos\omega t)$ and that instantaneous power applied is $\frac{a^2}{m\omega^2}\sin\omega t$.

Q.6. (a). A projectile is launched with an angle α from a cliff of height H above sea level. If it falls into area at a distance D from the base of the cliff, prove that maximum height above sea level is

$$H + \frac{D^2\tan\alpha}{4(H + D\tan\alpha)}$$

(b). A particle of unit mass describes an ellipse under the action of central force M r. Show that the normal component of the acceleration at any instant is $\frac{abM^{\frac{3}{2}}}{v}$, where v is the velocity at that instant and a, b are semi-axis of the ellipse.

Q.7. (a). Find the moment of inertia of a uniform solid sphere of mass m and radius a.

(b). A gun of mass M fires a shell of mass m horizontally and the energy of the explosion is such as would be sufficient to project the shell to height h . Show that the velocity of the recoil is

$$\sqrt{\frac{2m^2gh}{M(M+m)}}$$