KHYBER PAKHTOON KHWA, PUBLIC SERVICE COMMISSION, PESHAWAR

SUBJECT: COPETITIVE EXAMINATION FOR THE POSTS OF PROVINCIAL MANAGEMENT SERVICE (BPS-17)

PURE MATHEMATICS PAPER -1

Time Allowed: **03 hours** Max. Marks: 100

NOTE: Attempt **FIVE** questions in all, selecting at least **TWO** questions from each section. Extra attempt of question or part will not be considered. All questions carry equal marks.

SECTION – A

Q1(a): Define group and cyclic group. Let G be a cyclic group of order "n" generated by "a". Then for each positive divisor "d" of "n", there is a unique subgroup (of G) of order d.

(b): Find all the subgroups of a cyclic group of order 12.

Q2(a): Let G be a group and a, b ∈ G. Show that

(i) The order of **a** and a^{-1} are equal.

(ii) The order of **ab** and **ba** are equal.

(iii) The order of **a** and bab^{-1} are equal.

(b): Show that the set $S = \{\bar{1}, \bar{3}, \bar{5}, \bar{7}\}$ is a group under multiplication modulo 8. Find the order of each element of S.

Q3(a): The housing department of the KPK government plan to undertake four housing projects and lists of material requirements for each house in each of the project as follows:

	Project 1	Project 2	Project 3	Project 4
Paint(in 100 gallons)	1	2	1	2
Wood(in 10,000cu ft)	3	4	3	3
Bricks(in millions)	1	2	2	1
Labour(in 1000hrs)	10	10	9	8

If the supplier delivers 6,800 gallons paint, 1,420,000 cu fts of wood, 64 millions bricks and 4,48,000 hours of labour, find out the number of houses build for each project.

(b): State Cayley-Hamilton theorem and verify the theorem for the matrix M, if $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

Q4(a): If U and W are finite dimensional subspaces of a vector space V over a field F, then Dim (U + W) = dim U + dim W – dim (U ∩ W).

(b): Determine whether or not the given set of vectors is a basis for $R^3: \{(1,2,-1),(0,3,1),(1,-5\ 3)\}$.

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SECTION – B

Q5(a): Write the equation of the straight line passing through $(2,4,-1)$ and $(5,0,7)$.
Also find the direction cosines and direction ratios of this line.

(b): Show that the lines $x+3y-z+6=0; 2y+z=3$ and $\frac{x-3}{2}=\frac{y}{4}=\frac{1-z}{3}$ are perpendicular to each other. Also, find the acute angle between $\frac{x-3}{2}=\frac{y}{2}=\frac{z+1}{2}$ and $\frac{x}{3}=\frac{y+1}{-1}=\frac{z+2}{2}$.

Q6(a): Find the equation of the plane through (1,2,-1) and (2,-1,1) and perpendicular to the plane $3x-2y+z=7$.

(b): Find the equation of the plane passing through the intersection of planes $p_1 = 2x-y-3z=0$ and $x+2y-2z-3=0$, perpendicular to the plane $3x-2y+4z-6=0$,.

Q7(a): Find the total arc length of the cardioids $r = 1 - \cos\theta$.

(b): Convert from Rectangular to Cylindrical and spherical coordinates $(-2,2,0)$.

Q8(a): Find the equation of the sphere with its center on x=y=z and passing through (-1,4,1) and (5,3,0).

(b): The end point of the diameter of a sphere are $(-2,1,4)$ and $(6,3,-2)$. Find its equation.