K.P.K,PUBLIC SERVICE COMMISSION,PESHAWAR

COMPETATIVE EXAMINATION FOR PROVINCIAL MANAGEMENT SERVICE,2013

APPLIED MATHEMATICS,PAPER-I

TIME: 3 hours

Max Marks: 100

Note: Attempt only **FIVE** questions, selecting at least **TWO** questions from each section.

SECTION A

Q.1 (a) If $\vec{A} = 3xyz^2\hat{\imath} + 2xy^3\hat{\jmath} - x^2yz\hat{k}$ *and* $\varphi(x,y,z) = 3x^2 - yz$. Find (i) $\nabla.\vec{A}$ (ii) $\nabla.(\varphi\vec{A})$ (iii) $\vec{A}.\nabla\varphi$ (iv) $\nabla.\nabla\varphi$ at the point (1,-1,1). 08

(b) Prove that the Laplacian of $\frac{1}{r}$ is zero, where $r = \sqrt{x^2+y^2+z^2}$ 06

(c) If $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. Prove that $curl(\vec{a}\times\vec{r}) = 2\vec{a}$. Where $\vec{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$ is a constant vector. 06

Q.2 (a) If $\vec{F} = (2xy + z^3)\hat{\imath} + x^2\hat{\jmath} + 3xz^2\hat{k}$.Find the work done in moving an object in this field from (1,-2,1) to (3,1,4). 08

(b) Verify Green's theorem in the plane for $\oint(xy + y^2)dx + x^2dy$.Where the closed curve is bounded by $y = x$ *and* $y = x^2$. 06

(c) Evaluate

$$\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy \quad along\ the\ path\ \ x^4 - 6xy^3 = 4y^2$$ 06

Q.3 (a) Use stoke's theorem for $\vec{F} = (x+y)\hat{\imath} + (2x-z)\hat{\jmath} + (y+z)\hat{k}$ for the surface of a triangular shape with vertices (2,0,0),(0,3,0) and (0,0,6). 10

(b) Use triple integral to find the volume enclosed between $x^2 + y^2 = 9$ and the plane $z = 1$ *and* $x + z = 5$. 10

Q.4 (a) Find the centroid of the plane region bounded by parabola $y = 6x - x^2$ and the line $y = x$. 10

(b) A triangular lamina with vertices (0,0),(0,1) and (1,0) has the density function $\rho(x,y) = xy$ .Find its total mass and coordinates of centre of gravity. 10

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SECTION B

Q.5 (a) A projectile is launched at an angle θ so as to hit a target "T" which makes an angle $\alpha(\alpha < \theta)$ with the horizontal. Show that the range "R" is 10

$$\frac{2v_{0}.Cos\theta.Sin(\vartheta-\alpha)}{g.Cos^{2}\alpha}$$

(b) A ball is struck by a bat and 2 second later it is caught 20 meters away. 10
(i)If it was one meter above the ground when struck and caught, what was the greatest height it reached above the ground?
(ii)what was its speed ,when it was caught?

Q.6 (a) What are the conditions and characteristics of simple harmonic motion? Derive the expression for velocity and frequency, when the body executing simple harmonic motion. 10

(b) A body oscillates with simple harmonic motion according to the equation 10

$$X = (6.12\ m).Cos\left[\left(8.38\frac{rad}{sec}\right)t + 1.92\ rad\right]$$

Find the displacement, velocity and the acceleration at time $t = 1.90$ seconds

Q.7 (a) State and explain work energy theorem. What are the limitations of work energy theorem. 10

(b) A super ball can bounce 90% of its original length. 10
(i)How much energy is lost after a 30 gram ball is bounced once from an original height of 3 meters.
(ii)About how many bounces are required if the maximum height after the N^{th} bounce is 1% of original height.

Q.8 (a) Prove that if the total torque on a particle is zero the total angular momentum is constant. Also derive the relation between torque and angular momentum. 10

(b) Calculate the angular momentum of the earth that arises from its spinning motion on its axis and the angular momentum of the earth that arises from its orbital motion about the sun? Which is greater? 10
Given: Mass of earth$(5.98 \times 10^{24}kg)$
Radius of earth$(6.37 \times 10^{6}m)$

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