KPK, PUBLIC SERVICE COMMISSION

Competitive Examination for the posts of PMS, 2016

PURE MATHEMATICS , PAPER I

Time Allowed: 03 Hours   Max. Marks: 100

Instructions: Attempt **FIVE** questions in all. Select **THREE** from section A and **TWO** from section B.
All question carry equal marks.

SECTION A

Q1.(a) Define cyclic group? Find all the subgroups of a cyclic group of order 12. (10)

(b) Find all the subgroup of S_3? (10)

Q2.(a) Determine whether the following vectors linearly independent or not. (10)
$v_1(1,1,2)$, $v_2(1,2,5)$ *and* $v_3(5,3,4)$.

(b) Solve the system of equations. $x + 2y + z = 2$, $3x + y - 2z = 1$, $4x - 3y - z = 3$. (10)

Q3(a)Find the rank of the matrix $\begin{bmatrix} 2 & 2 & -1 & 0 & 1 & 0 \\ -1 & -1 & 2 & -3 & 1 & 0 \\ 1 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$. (10)

(b) Determine a basis for the null space of matrix $\begin{bmatrix} 1 & 2 & -3 & 2 & -3 \\ 1 & -1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 1 & -1 \\ 1 & 2 & 5 & -6 & -3 \end{bmatrix}$. (10)

Q4(a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$. (10)

(b) Use Cayley-Hamilton Theorem to find A^3 if $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. (10)

Section B

Q5(a) Find the equation of the plane passing through the points $(2,-3,1)$ and containing the line
$x - 3 = 2y = 3z - 1$. (10)

(b) Find the equation of the sphere if the centre is on the line $x = y = z$ and it passes through the points $(5,3,0)$ and $(-1,4,1)$. (10)

Q6(a) Find the equation of the tangent plane to the surface $z = x^2 + y^2$ at the point $(2,1,5)$. Find also the equation of normal line at that point. (10)

(b) Transform the equation of the curve $\rho = 3\cos\theta\sin\varphi$ into the spherical coordinates and rectangular coordinates. (10)

Q7(a) Determine the length of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ between the cusps. (10)

(b) Determine the curvature at the point $\left(\frac{1}{2}, \frac{1}{2}\right)$ on the folium $x^3 + y^3 = 3xy$.