Time Allowed: <u>03 Hours</u>   Max. Marks: <u>100</u>

<u>Instructions:</u> Attempt **FIVE** questions in all. Select **THREE** from section A and **TWO** from section B.
All question carry equal marks.

SECTION A

Q1(a). Evaluate the following: (i) $\lim_{x \to 0} (\cos x)^{cosec^2 x}$ (ii) $\lim_{x \to 0} \dfrac{e^{2x} - e^{-2x}}{\ln(1+x)}$ (10)

(b)   Discuss the differentiability of $f(x) = |x|$ at $x = 0$.   (10)

Q2(a) Find the asymptotes of the curve $y = \dfrac{x^3 + 2x - 1}{x}$.(10)

(b) Find the extreme values of the function $f(x, y) = y^2 + 4xy + 3x^2 + x^3$ (10)

Q3(a) Determine the volume of the solid bounded by planes $z = 0$, $z = 2 + x$ and the cylinder $x^2 + y^2 = 1$.   (10)

(b) Discuss the convergence of the series.   (10)

(i) $\displaystyle\sum_{n=1}^{\infty} \dfrac{n+1}{2n^2+1}$ (ii) $\displaystyle\sum_{n=1}^{\infty} \dfrac{2^n}{n(n+2)}$

Q4(a) Prove that $\dfrac{\pi}{4} < \displaystyle\int_0^{\frac{\pi}{4}} \sec x \; dx < \dfrac{\pi}{\sqrt{8}}$.   (10)

(b) Evaluate the following by using Beta and Gamma integrals.   (10)

(i) $\int_0^1 t^9 (1 - t^2)^7 dt$ (ii) $\int_0^{\infty} e^{-x} x^5 \; dx$

SECTION B

Q5(a) Simplify $\left(\dfrac{\sqrt{3}-i}{\sqrt{3}+i}\right)^6$ using De Moivre's Theorem.   (10)

(b) Let $f(z) = u(x, y) + i \, v(x, y)$ be an analytic function. If $u(x, y) = 3x - 2xy$, then find $v(x, y)$ and express $f(z)$ in terms of z.   (10)

Q6(a) Evaluate $\int_c \dfrac{1-2z}{z(z-1)(z-2)} \; dz$ where c is the circle $|z| = 1.5$.   (10)

(b) Evaluate by Residue Theorem $\int_0^{2\pi} \dfrac{\cos 2\theta}{5 + 4\cos\theta} \; d\theta$.   (10)

Q7(a) Find a fourier series to represent $f(x) = x^2 + x$ from $-\pi$ to π and show that   (10)

$\dfrac{\pi^2}{6} = 1 + \dfrac{1}{2^2} + \dfrac{1}{3^2} + \dfrac{1}{4^2} + \cdots$

(b) Find the fourier transform of the function $f(x) = \begin{cases} 1 & , \; |x| < a \\ 0 & , \; |x| > a \end{cases}$ (10)